

### **Arithmetic Formulaes :**

1. For the numbers in arithmetic progression,

N'th terms:  $u_n = a + (n - 1)d$

Where a = first terms, d = common difference.

N'th mean:  $X_n = a + \frac{n(b - a)}{n - 1}$

2. For the arithmetic series,

The last term  $(l) = a + (n - 1)d$

The sum,  $S_n = \frac{1}{2}n\{2a + (n - 1)d\} = \frac{1}{2}n(a + l)$

3. Sum of squares of n natural numbers  $S_n = \frac{n(n + 1)(2n + 1)}{6}$

4. Sum of cubes of n natural numbers  $S_n = \left\{\frac{n(n + 1)}{2}\right\}^2$

5. For the numbers in geometric progression,

N'th term:  $u_n = ar^{n-1}$  and n'th mean,  $X_n = a \left(\frac{b}{a}\right)^{\frac{n}{n+1}}$

### **Geometric Series**

1. For the geometric series, the sum

$$S_n = \frac{a(r^n - 1)}{r - 1} \text{ if } r > 1, \quad S_n = \frac{a(1 - r^n)}{1 - r} \text{ if } r < 1$$

$$S_n = \frac{a - rl}{1 - r} \text{ if } r < 1, \quad S_n = \frac{a}{1 - r} \text{ if } n \text{ is very large,}$$

2. For any two numbers  $a$  and  $b$ .

$$\frac{a + b}{2} \geq \sqrt{ab} \text{ if } a \neq b, \quad \frac{a + b}{2} = \sqrt{ab} \text{ if } a = b$$

3. Mean

$$\text{i. a. Arithmetic mean (A.M.)} = \frac{a + b}{2}$$

b. For more arithmetic means "d" is to be found as  $d = \frac{b - a}{n + 1}$  then,  
mean

$$M_1 = a + d \quad M_2 = a + 2d \quad M_3 = a + 3d \dots \dots \dots$$

Where  $a$  = first term,  $b$  = last term,  $d$  = common difference

$$\text{ii.a. Geometric mean (G.M.)} = \sqrt{ab}$$

b. For more geometri means "r" is to be found as

$$r = \left(\frac{b}{a}\right)^{\frac{1}{n + 1}} \text{ then, mean}$$

$$M_1 = ar \quad M_2 = ar^2 \quad M_3 = ar^3 \dots \dots$$

Where  $a$  = first term,  $b$  = last term,  $r$  = common ratio.

4. If AM & GM be the A.M. and G.M. between two positive numbers  $a$  and  $b$  then  $AM \geq GM$ .

5. Sum of the first 'n' natural numbers (  $1 + 2 + 3 + \dots + n$  )  
is  $S_n = \frac{n}{2}(n + 1)$ .

6. Sum of the first 'n' odd natural numbers (  $1 + 3 + 5 + \dots$  to n terms )  
is  $S_n = n^2$

7. Sum of the first 'n' even natural numbers (  $2 + 4 + 6 + \dots$  to n terms )  
is  $S_n = n(n + 1)$

8. Sum of squares of the first 'n' natural numbers

(  $1^2 + 2^2 + 3^2 + \dots + n^2$  ) is  $S_n = \frac{1}{6}n(n + 1)(2n + 1)$ .

9. Sum of the cubes of first 'n' natural numbers

(  $1^3 + 2^3 + 3^3 + \dots + n^3$  ) is  $S_n = \left\{ \frac{n(n + 1)}{2} \right\}^2$ .

10.  $r = \frac{t_n}{t_{n-1}}$

11.  $t_n = ar^{n-1}$

## **Simple Interest Formulas:**

1. To calculate the total simple interest (S.I.) on a principle amount (P) at a certain rate per annum ( R), after a certain years of time(T) is calculated using formula:

$$S.I. = \frac{P \times T \times R}{100}$$

2. If Amount (A) is the total amount to be paid to the lender at the end of lending term , Principal (P) is the total money lend from the lender , Interest (I) is the total interest accumulated , Rate of Interest (R) is the rate of interest per annum and Time (T) is the total time in years for which money is lend then,

$$A = P + I = P \times \left( \frac{100 + RT}{100} \right)$$

3. If "P" is the total principal or money lend , "A" is the total amount to be paid to lender at the end of lending term , "R" is the rate of interest per annum and "T" is total lending term in years then:

$$P = \frac{100 \times A}{RT + 100}$$

4. When , "T" is total lending term in years , "I" is the total Simple Interest accumulated in time "T" , "P" is the total principal lend and "R" is the rate of interest per annum then:

$$T = \frac{100 \times I}{PR}$$

5. To find the Rate of interest We can use following formula:

$$R = \frac{100 \times I}{PT}$$

Where, "R" is rate of interest , "I" is interest rate , "P" is total principal and "T" is total time.

6. We can also calculate Principal amount by using formula:

$$P = \frac{100 \times I}{TR}$$

Where, "I" is rate of interest , "T" is total , "R" is rate of interest.

Please note that , In all of the above formulas , The unit of rate of interest (I) and Time (T) should be corresponding , for example if "R" is the rate on interest per month then "T" should be in months , If "R" is rate of interest per annum then "T" should be in Years.

1> **Derivative of Constant function or derivative of  $f(x)=y=c$  (c is a constant)**

Let  $\Delta x$  be a small increment in  $x$  and  $\Delta y$  be corresponding increment in  $y$ . Then,

$$f(x + \Delta x) = y + \Delta y = c$$

$$\text{or, } \Delta y = c - y$$

$$= c - c$$

$$= 0$$

$$\text{and, } \frac{\Delta y}{\Delta x} = 0$$

$$\text{Thus, } \frac{dy}{dx} = \lim_{\Delta x \rightarrow 0} \frac{\Delta y}{\Delta x} = 0$$

2> **Derivative of Identity function or derivative of  $f(x)=y=x$**

Let  $\Delta x$  be a small increment in  $x$  and  $\Delta y$  be corresponding increment in  $y$ . Then,

$$f(x + \Delta x) = y + \Delta y = x + \Delta x$$

$$\text{or, } \Delta y = x + \Delta x - y$$

$$= \Delta x$$

$$\text{and, } \frac{\Delta y}{\Delta x} = 1$$

$$\text{Thus, } \frac{dy}{dx} = \lim_{\Delta x \rightarrow 0} \frac{\Delta y}{\Delta x} = 1$$

3> **Derivative of simple Quadratic function or derivative of  $f(x)=y=x^2$**

Let  $\Delta x$  be a small increment in  $x$  and  $\Delta y$  be corresponding increment in  $y$ . Then,

$$f(x + \Delta x) = y + \Delta y = (x + \Delta x)^2$$

$$= x^2 + 2.x.\Delta x + \Delta x^2$$

$$\text{or, } \Delta y = x^2 + 2.x.\Delta x + \Delta x^2 - y$$

$$= \Delta x(2x + \Delta x)$$

$$\frac{\Delta y}{\Delta x} = 2x + \Delta x$$

and,

$$\frac{dy}{dx} = \lim_{\Delta x \rightarrow 0} 2x + \Delta x = 2x$$

Thus,

#### 4> Derivative of Simple cubic function or derivative of $f(x)=y=x^3$

Let  $\Delta x$  be a small increment in  $x$  and  $\Delta y$  be corresponding increment in  $y$ . Then,

$$f(x + \Delta x) = y + \Delta y = (x + \Delta x)^3$$

$$y + \Delta y = x^3 + 3x^2.\Delta x + 3x.\Delta x^2 + \Delta x^3$$

$$\text{or, } \Delta y = x^3 + 3x^2.\Delta x + 3x.\Delta x^2 + \Delta x^3 - y$$

$$= 3x^2.\Delta x + 3x.\Delta x^2 + \Delta x^3$$

$$= \Delta x(3x^2 + 3x.\Delta x + \Delta x^2)$$

$$\frac{dy}{dx} = 3x^2 + 3x.\Delta x + \Delta x^2$$

and,

Thus,

#### The Conclusion:

If we analysis above four examples and also analysis the derivative of higher degree of functions

Then we can see the following result:

Derivative of simple algebraic functions or polynomial function

like function  $f(x)=y=x^n$

is  $n.x^{n-1}$

$$\frac{dx^n}{dx} = n.x^{n-1}$$

or,





**Derivative:**

When a variable "y" is defined as a function of another variable "x" or,

$$f(x)=y$$

Then , The Derivative or Differential Coefficient of the function "f" at a point "x" or with respect to "x" is the limiting value of:

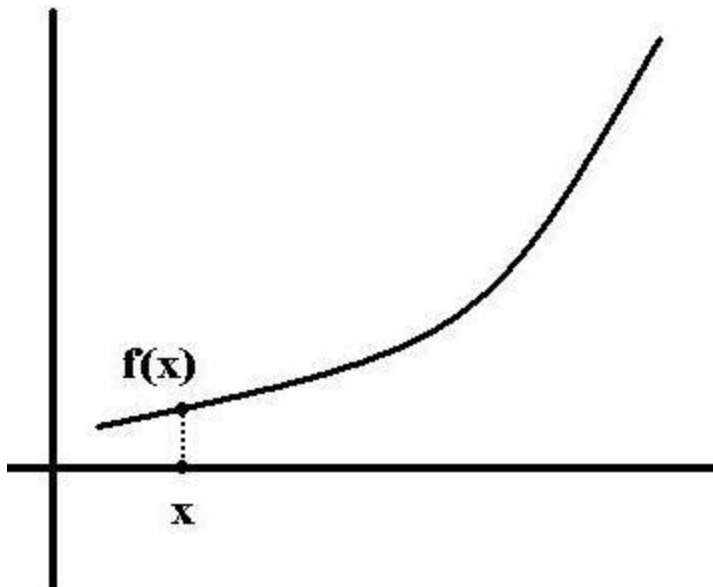
$$\lim_{x \rightarrow 0} \frac{f(x + \Delta x) - f(x)}{\Delta x}$$

The derivative of a function of "x" with respect to "x" is denoted by:

$$\frac{df(x)}{d(x)}$$

for example:

If "y" is a function of "x" or  $f(x)=y$  whose graph looks like:



Then the derivative of the function "f" with respect to "x" at point "x" is :-

which can be shown in figure as:

